

C.U.SHAH UNIVERSITY

Summer Examination-2017

Subject Name : Engineering Mathematics-II

Subject Code : 4TE02EMT2

Branch: B.Tech (All)

Semester : 2

Date :04/05/2017

Time : 02:00 To 05:00

Marks : 70

Instructions:

- (1) Use of Programmable calculator & any other electronic instrument is prohibited.
- (2) Instructions written on main answer book are strictly to be obeyed.
- (3) Draw neat diagrams and figures (if necessary) at right places.
- (4) Assume suitable data if needed.

Q.1 Attempt the following questions:

(14)

a) $\int_0^{\frac{\pi}{2}} \cos^7 \theta \, d\theta = \underline{\hspace{2cm}}$.

(a) 0 (b) $\frac{16}{35}$ (c) $\frac{32}{35}$ (d) $\frac{8\pi}{35}$.

b) If $f_n = \int_0^{\frac{\pi}{4}} \tan^n \theta \, d\theta$ then $f_n + f_{n-2} = \underline{\hspace{2cm}}$.

(a) $\frac{1}{n}$ (b) $\frac{1}{n-1}$ (c) $\frac{n}{n-1}$ (d) $\frac{n-1}{n}$.

c) State the Euler's formula.

d) $\Gamma(4.5) = \underline{\hspace{2cm}}$.

(a) $4.5 \Gamma(3.5)$ (b) $3.5 \Gamma(4)$ (c) $4 \Gamma(3.5)$ (d) none of these.

e) $\beta\left(\frac{1}{2}, \frac{1}{2}\right) = \underline{\hspace{2cm}}$.

(a) $\sqrt{\pi}$ (b) $\frac{1}{2}$ (c) 1 (d) π .

f) Define a complete elliptical integral of 1st kind.

g) $\operatorname{erf}(x) + \operatorname{erfc}(x) = \underline{\hspace{2cm}}$.

(a) 0 (b) 1 (c) 2 (d) none of these.

h) The curve $y^2(2a-x) = x^3$ is symmetrical about _____.

(a) X-axis (b) Y-axis (c) origin (d) line $y = x$.

i) The length of the spiral $r = e^{\theta/\sqrt{2}}$, $0 \leq \theta \leq \pi$, is _____.

(a) e^{π} (b) $e^{\pi} + 1$ (c) $e^{\pi} - 1$ (d) 1.

j) $\int_0^{\pi} \int_0^x x \sin y \, dy \, dx = \underline{\hspace{2cm}}$.

(a) $\pi/2$ (b) $\pi^2/2$ (c) $(\pi/2) + 1$ (d) $(\pi^2/2) + 1$.



- k) $\int_0^1 \int_0^1 \int_0^1 (x^2 + y^2 + z^2) dz dy dx = \underline{\hspace{2cm}}$.
 (a) 1 (b) -1 (c) 0 (d) none of these.
- l) The series $\sum_{n=0}^{\infty} \frac{(-1)^n}{2^n}$ is _____.
 (a) convergent (b) divergent (c) conditionally convergent
 (d) none of these.
- m) The series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ is divergent if _____.
 (a) $p > 1$ (b) $p \leq 1$ (c) $p \geq 1$ (d) $p < 1$.
- n) The differential equation $(x + y) dy + (x - y) dx = 0$ is _____ differential equation.

Attempt any four questions from Q-2 to Q-8

Q.2 Attempt all questions **(14)**

a) Evaluate: **(02)**

$$\int_0^{\pi} (1 + \cos \theta)^4 d\theta$$

b) Evaluate: **(02)**

$$\int_0^{\frac{1}{2}} x^3 \sqrt{1 - 4x^2} dx$$

c) Evaluate: **(04)**

$$\int_0^1 \frac{x^6}{(1 + x^2)} dx$$

d) Trace the curve $x^3 + y^3 = 3axy$. **(06)**

Q.3 Attempt all questions **(14)**

a) If $\beta(n, 3) = 1/105$ and n is a positive integer, then find n . **(02)**

b) Prove that: (i) $n\beta(m + 1, n) = m\beta(m, n + 1)$; and **(04)**
 (ii) $\beta(m, n) = \beta(m, n + 1) + \beta(m + 1, n)$.

c) Evaluate: **(04)**

$$\int_{-\infty}^{\infty} e^{-k^2 x^2} dx$$

d) Prove that: **(04)**

$$\int_0^{\infty} \frac{x^4}{4^x} dx = \frac{24}{(\log 4)^5}$$



Q.4 Attempt all questions (14)

a) Prove that:

$$\int_0^{\infty} \frac{\sqrt{x}}{x^2 + 2x + 1} dx = \frac{\pi}{2} \quad (03)$$

b) Evaluate:

$$\int_0^{\frac{\pi}{3}} \frac{d\theta}{\sqrt{3 - 4\sin^2\theta}} \quad (03)$$

c) If the perimeter of the ellipse of $e = 1/\sqrt{2}$ is equal to twice the length of one arch of the curve $y = \sin x$, then find the area of the ellipse. (04)

d) (1) Show that $\operatorname{erf}(x)$ is an odd function; and (04)

(2) prove that:

$$\int_{-a}^a e^{-t^2} dt = \sqrt{\pi} \operatorname{erf}(a)$$

Q.5 Attempt all questions (14)

a) Trace the curve $r^2 = a^2 \cos 2\theta$. (04)

b) Find the length of the Cardioid $r = 1 + \cos \theta$. (02)

c) Find the area of the smaller region lying above X-axis and bounded by the circle $x^2 + y^2 = 2x$ and the parabola $y^2 = x$. (04)

d) Find volume of the solid generated by revolving the lemniscate $r^2 = a^2 \cos 2\theta$ about the line $\theta = \pi/2$. (04)

Q.6 Attempt all questions (14)

a) Evaluate:

$$\iint_R (x^2 + y^2) dA \quad (02)$$

, where R is a triangular region with vertices (0, 0), (0, 1) and (1, 0).

b) Evaluate: (03)

$$\int_0^1 \int_y^1 x^2 e^{xy} dx dy$$

c) Evaluate: (03)

$$\int_{-1}^1 \int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} \ln(x^2 + y^2 + 1) dx dy$$

d) Find the volume of the region D between the cylinder $z = y^2$, and the XY-plane that is bounded by the planes $x = 0$, $x = 1$, $y = -1$, $y = 1$. (03)

e) Evaluate: (03)

$$\int_0^1 \int_{\sqrt{x}}^1 \int_0^{\ln 3} \frac{\pi e^{2x} \sin(\pi y^2)}{y^2} dx dy dz$$



Q.7 Attempt all questions (14)

a) Discuss convergence/divergence of the following series:

i) $\sum_{n=1}^{\infty} \frac{2n+1}{n^2(n+1)^2}$ ii) $\sum_{n=1}^{\infty} \frac{1}{2\sqrt{n+3}\sqrt[3]{n}}$ iii) $\sum_{n=1}^{\infty} \frac{n^{10}}{10^n}$ iv) $\sum_{n=1}^{\infty} (-1)^n \ln\left(1 + \frac{1}{n}\right)$ (08)

b) Prove that if the series $\sum a_n$ converges, then (02)

$$\lim_{n \rightarrow \infty} a_n = 0$$

Find the values of x for which the following power series converges. (04)

c) $\sum_{n=1}^{\infty} (-1)^n \frac{(x+2)^n}{n}$

Q.8 Solve the following ordinary differential equations: (14)

a) $(x - y)dx - (x + y)dy = 0$ (03)

b) $(y - x^3)dx + (x + y^3)dy = 0$ (03)

c) $(1 + x^2)dy + 2xy dx = \cot x dx$ (04)

d) $2yy'' = 1 + (y')^2$ (04)

